CERN-TH/95-71

DFTT 95/23

ANOMALOUS EVOLUTION EFFECTS ON SINGLET STRUCTURE FUNCTIONS

Marco Genovese

Theory Division, CERN, CH-1211 Geneva 23, Switzerland
Dipartimento di Fisica Teorica, Università di Torino,
and INFN, Sezione di Torino, Via P.Giuria 1, I-10125 Turin, Italy.

Abstract

A study of the effects of the anomalous evolution due to mesonic degrees of freedom on singlet structure functions is presented. Possible phenomenological applications are discussed.

CERN-TH/95-71

March 1995

One of the most successful results of perturbative QCD is surely the detailed description of logarithmic scaling violations in Deep Inelastic Scattering (DIS). Recent improvements in experimental accuracy are now opening up the possibility of measuring subleading perturbative corrections, as well as corrections to perturbation theory as expressed by higher twist effects. However, it was pointed out in [1] that in the region of $Q^2 \sim 1$ $(\mathrm{GeV/c})^2$ the scale dependence of the non-singlet nucleon structure function may be qualitatively rather different from that predicted by purely perturbative QCD, due to non-perturbative effects related to the anomalous breaking of axial U(1) symmetry.

In [2] the effects on non-singlet structure functions evolution were studied, producing testable predictions of this model. This predictions are in good qualitative agreement with the experimental data available today and could lead to a clear confirmation of the model, when more precise data will be available.

Here we will study the effects on the singlet structure function evolution, which are expected in this scheme [1].

Let us first summarize the main features of the formalism [1].

In the model of [1] is introduced, beyond the usual Altarelli-Parisi [5] QCD evolution, the radiation of bound states (pseudoscalar mesons) by quarks. This leads to modify the evolution equations, adding new splitting functions, functions that describe this phenomenon.

In Ref. [1] the mesons ($\Pi(k)$) are coupled to quarks by the effective action

$$S_{\rm eff} = \int d^4p \; \bar{\psi}(p) i \not{D} \psi(p) + \int d^4p \int d^4k \; \bar{\psi}(p+\tfrac{1}{2}k) f_\Pi \chi(k,p) U(k) \psi(p-\tfrac{1}{2}k), \qquad (1)$$

where $U(k) \equiv \exp\left(i\gamma_5 \Pi(k)/f_\Pi\right)$, f_Π is the meson decay constant and the coupling is defined by

$$\chi(k,p) \equiv S^{-1}(p+\tfrac{1}{2}k)\langle 0|\psi(p-\tfrac{1}{2}k)\bar{\psi}(p+\tfrac{1}{2}k)|\Pi(k)\rangle S^{-1}(p-\tfrac{1}{2}k), \qquad (2)$$

where $S^{-1}(p) = (\not{p} + \Sigma(p^2))^{-1}$ is the quark propagator ($\Sigma(p^2)$ is the self-energy).

[1] Incidentally, a previous tentative qualitative estimate of these non-perturbative effects on singlet evolution from the results obtained in the non-singlet case had led to the suggestion [3] that the effect could be opposite in sign and larger than the one we calculate here, giving a possible explanation to the albeit marginal difference between the α_s extracted by DIS and LEP data [4]. This quantitative analysis does not confirm the earlier qualitative estimate.

The effective coupling $\chi(k,p)$ is then expanded, in the most general case, in terms of four vertex functions (a pseudoscalar, two axials and a tensor.)

This permits us to evaluate the cross section $\sigma_{q_i q_j}^{\gamma^* X}(x;t)$ for the absorption of a virtual photon γ^* and the emission of a pseudoscalar meson X and, then, the splitting function

$$\left[\mathcal{P}_{q_i q_j}(x;t)\right]_X = \frac{d}{dt}\sigma_{q_i q_j}^{\gamma^* X}(x;t). \tag{3}$$

For the sake of simplicity, considering that the mixing angle is small (see, for example, [6]), we assume in the following that η and η' are the pure octet and singlet respectively. One obtains for the non-diagonal ($\mathcal{P}_{qq}^{ND}$, flavour changing) and for the diagonal ($\mathcal{P}_{qq}^{D}$) splitting functions:

$$\begin{aligned} \mathcal{P}_{ud}^{ND} &= \mathcal{P}_{du}^{ND} = \frac{d}{dt}\sigma^{\gamma^*\pi^+} = \frac{d}{dt}\sigma^{\gamma^*\pi^-} \\ \mathcal{P}_{uu}^{D} &= \mathcal{P}_{dd}^{D} = \frac{d}{dt}\left(\tfrac{1}{2}\sigma^{\gamma^*\pi^0} + \tfrac{1}{6}\sigma^{\gamma^*\eta} + \tfrac{1}{3}\sigma^{\gamma^*\eta'}\right). \end{aligned} \tag{4}$$

The strange mesons (K) contributions are kinematically suppressed and therefore neglected. In the following we will thus neglect anomalous contributions to the strange sea evolution.

In [2] the distribution

$$q^+(x) \equiv u(x) + \bar{u}(x) - d(x) - \bar{d}(x) \tag{5}$$

was studied.

For this the (mesonic) non-singlet evolution is given by

$$\frac{d}{dt}(u-d)^+ = (\mathcal{P}_{qq}^{D} - \mathcal{P}_{qq}^{ND}) \otimes (u-d)^+. \tag{6}$$

Because of the strong mass difference between η' ($m_{\eta'} = 958$ MeV$/c^2$) and pions ($m_{\pi^0} = 135$ MeV$/c^2$), which is caused by $U(N_f)$ breaking due to axial anomaly [7], there is a relevant difference between $\mathcal{P}_{qq}^{D}$ and $\mathcal{P}_{qq}^{ND}$, which is responsible of quark-antiquark sea $SU(2)_F$ symmetry breaking.

The fact that the $U(N_f)$ breaking concerns pseudoscalar mesons only implies that these bound states only contribute to non-singlet evolution: for other mesons' multiplets, $\mathcal{P}_{qq}^{D}$ and $\mathcal{P}_{qq}^{ND}$ are almost equal and moreover they are kinematically suppressed (of about one order of magnitude) due to the larger masses of these mesons.

The explicit calculation of cross sections $\sigma_{q_i q_j}^{\gamma^* X}(x;t)$ [1] shows that they depend on scalar (φ, which dominates at intermediate Q^2) and axial ($\tilde{\varphi}$, which determines the Q^2 tail) vertex functions only.

A dipole form is then assumed for the vertex functions (p is the quark momentum, $f_\pi = 93$ MeV)

$$\varphi(p^2) = \frac{m_d}{f_\pi} \frac{\Lambda^2 + m_d^2}{\Lambda^2 + p^2}, \tag{7}$$

$$\tilde{\varphi}(p^2) = \frac{g_\pi}{f_\pi} \frac{\tilde{\Lambda}^2 + m_d^2}{\tilde{\Lambda}^2 + p^2}, \tag{8}$$

in terms of the three parameters Λ, $\tilde{\Lambda}$ and g_π. In [1] $\tilde{\Lambda} = \Lambda$ is hypothesized (the dependence of the results on the precise value of $\tilde{\Lambda}$ is small anyway), while g_π is between 0 and 1.

The Λ value may be related, using chiral Ward identities, to the constituent quark mass. This requires it to vary in the interval $0.4\ \mathrm{GeV}/c^2 \lesssim \Lambda \lesssim 0.8\ \mathrm{GeV}/c^2$. The Λ and g_π values have been fixed in Refs. [1,2] to $\Lambda = 0.4\mathrm{GeV}/c^2$ and $g_\pi = 0.5$ by the request of reproducing the NMC Gottfried sum datum [8].

In [1] it has also been calculated that the anomalous dimension due to pion emission is significantly different from zero in the region $0.05\ (\mathrm{GeV}/c)^2 \lesssim Q^2 \lesssim 5$–$10\ (\mathrm{GeV}/c)^2$. The evolution must therefore be extended up to very low Q^2.

The strategy of [2] was to insert the effect of the meson emission inside the model of Ref. [9], which permits an extension to the Q^2 evolution up to very low Q^2 in order to study the non-singlet evolution starting from a reasonable ansatz at $Q^2 \approx 0(\mathrm{GeV}/c)^2$.

In the following we will adopt a different strategy in order to study the anomalous singlet evolution, namely we will begin the evolution at an input scale Q_0^2 sufficiently high to allow other non-perturbative effects on Q^2 evolution to be neglected a part from the meson emission ones (in the following we will assume as a reasonable choice $Q^2 \gtrsim 1$–$1.5\ (\mathrm{GeV}/c)^2$). This allows us also to use as input an ordinary parametrization of the experimental structure functions.

As we are evaluating a first approximation of these effects, we will neglect (as in [1,2]) the effects due to mesons other than the pseudoscalar ones, which are expected to be kinematically suppressed due to larger masses.

For the singlet evolution one has to consider the full set of coupled equations (omitting gluon contributions for simplicity):

$$
\begin{aligned}
\frac{d}{dt} q_i &= \sum_j \mathcal{P}_{q_i q_j} \otimes q_j + \sum_j \mathcal{P}_{q_i \bar{q}_j} \otimes q_j + \sum_a \mathcal{P}_{q_i \Pi^a} \otimes \Pi^a , \\
\frac{d}{dt} \bar{q}_i &= \sum_j \mathcal{P}_{\bar{q}_i \bar{q}_j} \otimes \bar{q}_j + \sum_j \mathcal{P}_{\bar{q}_i q_j} \otimes q_j + \sum_a \mathcal{P}_{\bar{q}_i \Pi^a} \otimes \Pi^a , \\
\frac{d}{dt} \Pi^a &= \sum_j \mathcal{P}_{\Pi^a q_j} \otimes q_j + \sum_j \mathcal{P}_{\Pi^a \bar{q}_j} \otimes \bar{q}_j + \sum_b \mathcal{P}_{\Pi^a \Pi^b} \otimes \Pi^b ,
\end{aligned}
\tag{9}
$$

where $\Pi^a(x;t)$ are the distribution functions of mesons of the pseudoscalar multiplet.

The distributions must fulfill the momentum sum rule

$$
\int_0^1 dx \cdot x \cdot \left[\sum_j (q_j + \bar{q}_j) + \sum_a \Pi_a + g \right] = 1 \tag{10}
$$

where g is the gluon distribution.

The first two equations in (9) express the evolution of the quark and antiquark distribution due to mesons emission, whereas the last equation gives the evolution of the pseudoscalar meson distribution. The former are determined by the splitting functions $\mathcal{P}_{qq}$, which expresses the probability of a quark to be emitted by another quark (with emission of an unobserved meson), and $\mathcal{P}_{q\Pi}$, which expresses the probability of a pseudoscalar to fragment into a quark and an antiquark (one of which is observed); at first order $\mathcal{P}_{q\bar{q}}$ is vanishing. The latter is determined by the splitting functions $\mathcal{P}_{\Pi q}$ and $\mathcal{P}_{\Pi\,\Pi}$, which give the probability of a pseudoscalar to be emitted by a quark or another pseudoscalar, respectively.

It must be noticed that not all these splitting functions are independent, being related by isospin and charge-conjugation invariance (see [1]).

Considering that we will be interested in the medium-to-large x region (far from the low x region where one expects a rapid growth of the singlet structure function due to the radiative generation of the sea from the glue), we will neglect, as a first approximation, the effect of the splitting of the radiated mesons in a $q\bar{q}$ pair, which is expected to contribute substantially to the low x region only and that is in any case also suppressed by the fact that the mesons distributions are not expected to be very large [2].

We will impose the momentum conservation, adding, as usual [10], a delta contribution to the splitting functions:

$$P^D(z) = \mathcal{P}^D(z) - \mathcal{P}_\Delta \delta(z-1) \tag{11}$$

and

$$P^{ND}(z) = \mathcal{P}^{ND}(z) - \mathcal{P}_\Delta \delta(z-1), \tag{12}$$

where

$$\mathcal{P}_\Delta = \int_0^1 dz \cdot z \cdot (\mathcal{P}^D(z) + \mathcal{P}^{ND}(z)). \tag{13}$$

The fact that we have neglected the mesonic distributions is thus phenomenologically compensated, in order to keep the validity of the momentum sum rule (10), by the explicit choice of the value of $\mathcal{P}_\Delta$.

The F_2 evolution is then given by:

$$\begin{aligned}\frac{dF_2(x,t)}{dt} = \int_x^1 \frac{dy}{y} &\left[P_{AP}(x/y) + P^D(x/y) + P^{ND}(x/y)\right] \times \\ &\left[4/9 \cdot (u(y) + \bar{u}(y)) + 1/9 \cdot (d(y) + \bar{d}(y))\right] + \\ &P_{AP}(x/y) \times 1/9 \cdot [s(y) + \bar{s}(y)]\,,\end{aligned} \tag{14}$$

where the mesonic effects concern the u and d quarks only (as we are neglecting the kinematically suppressed contributions of strange mesons) and P_{AP} are the usual Altarelli-Parisi splitting functions [5]. The heavy quarks, considering the low Q^2 under investigation, are completely negligible.

The numerical calculations are made starting with an input for the different parton distributions at $Q_0^2 = 1.6(\mathrm{GeV}/c)^2$; in the following we will show the results obtained using as input the CTQE3M distributions [11]; however, we have explicitly checked that our results are largely independent from this choice.

We carry out the evolution (14) through many small steps in Q^2 (we have explicitly verified the stability of our results on the choice of the steps), adding at every little step in Q^2, the mesonic contribution to the one coming from the Altarelli-Parisi part (at leading order). This last part of the evolution is effected in the momentum space (in order to have a better numerical accuracy); therefore, at each Q^2 step, the singlet, non-singlets and glue distributions are parametrized in the form $p_1 \cdot x^{p_2} \cdot (1-x)^{p_3} \cdot (1 + p_4 x^{0.5} + p_5 x + p_6 x^2 + p_7 x^3)$, which permits an analytical Mellin transform to the momentum space. The inverse Mellin transform to x space is made numerically. The value $\Lambda_{LO}^{nf=3} = 200$ MeV/c is used.

The mesonic effects tend to increase the growth of the parton distributions at low x (under $x \approx 0.15$), while at larger x the effect of the delta term dominates producing a more negative $\frac{d \log F_2}{dt}$. Altogether this therefore simulates a larger value of $\alpha_s(Q^2)$, except for a very narrow region around the point where the mesonic contribution changes of sign.

In fig. 1 $\frac{d \log F_2}{dt}$ is shown, as obtained with (solid curve) and without (dashed curve) mesonic effects at $Q^2 = 1.6(\mathrm{GeV}/c)^2$.
For $x \simeq 0.6$–0.7 there is a difference of 0.03 ($\sim$ 10%) between the two predictions (at higher x non-leading twist effects, as target mass corrections, may be quite important). At this scale the mesonic effects simulate an increase of $\Lambda_{LO}^{nf=3}$ from 200 MeV/c to 210–220 MeV/c. At larger scales the effect reduces rapidly. It is already small at $Q^2 \simeq 5$ $(\mathrm{GeV}/c)^2$ (where it is reduced by about one half of its former value), albeit always leading to an overestimate of Λ_{QCD} of $\approx 5\%$, and it is negligible for $Q^2 \simeq 10$ $(\mathrm{GeV}/c)^2$.

It is thus not yet really possible to observe this effect in today's DIS data; anyway the order of magnitude of the effect at $Q^2 \approx 1$ $(\mathrm{GeV}/c)^2$ is such that it could constitute a non completely negligible source of error in the determination of α_s by DIS data. A precise determination of this error obviously depends on the weights assumed by different Q^2 and x regions in a global fit; however, one can roughly expect a possible overestimate of Λ_{QCD} of a $\approx$ 5–10% for a global fit in a region such as 1 $(\mathrm{GeV}/c)^2$ $\lesssim Q^2 \lesssim 5$ $(\mathrm{GeV}/c)^2$ and $x \gtrsim 0.01$.

When higher precision experimental data will become available it would surely be quite interesting to search for a confirmation of this effect by comparing $\frac{d \log F_2}{dt}$ evaluated with Λ_{QCD} obtained in other processes and the one extracted by DIS data in the $Q^2 \lesssim 5(\mathrm{GeV}/c)^2$ region. However, a direct observation of this effect could turn out to be rather difficult due to the fact that scale fixing and other theoretical uncertainties could lead to an arbitrariness in determining a precise Λ_{QCD} value, which is of the same order as this effect.

The study of the singlet anomalous evolution enables us to evaluate the anomalous evolution effects on $\frac{d(F_2^n/F_2^p)}{dt}$ also (where F_2^n is the neutron structure function and F_2^p the proton one.)

It must be noticed that the numerical estimate $\frac{d(F_2^n/F_2^p)}{dt}$ is much more sensitive to the choice of the parton distributions than the evaluation of the derivative of F_2. The estimate of the effect of the mesonic contributions to this quantity is thus, for a precise numerical calculation, partially depending on this choice as well. Anyhow for a first evaluation of the effect this dependence is not really severe, as we have checked with other inputs beyond the

CTEQ3M parton distributions (used as input at $Q^2 = 1.6(\mathrm{GeV}/c)^2$ as in the F_2 derivative case).

In this case the mesonic effect is a positive contribution to $\frac{d(F_2^n/F_2^p)}{dt}$ in the whole range of x. It enhances this quantity with respect to the one evaluated with only the Altarelli-Parisi splitting functions of ≈ 0.015 at $Q^2 = 1.6(\mathrm{GeV}/c)^2$ in the region $0.1 < x < 0.6$. The effect is still larger at higher x, where a careful analysis of target mass effects and of other higher twist effects would anyway be necessary, while it disappears at lower x (at $x = 0.01$ it is reduced to a difference of 0.003).

The effect decreases with Q^2 and it is reduced by 50% at $Q^2 = 10(\mathrm{GeV}/c)^2$.

Nowadays experimental data [12] are not yet able to discern this effect because of large errors. Anyway they are in qualitative agreement with this prediction for $x \approx 0.4$–0.6, where they show values larger than zero, above QCD predictions, while they seem to point out more negative values than the QCD prediction for $x \approx 0.1$–0.3. In [12] this effect has been interpreted in terms of higher twist effects.

More precise experimental data and a more careful treatment of higher twist effects (for example by fitting them carefully in high precision experimental data or by evaluating them in some theoretical scheme) could lead in a next future to an at least qualitative test of this prediction.

In summary we have presented an estimate of the effects of the anomalous evolution due to the mesonic degrees of freedom on singlet structure functions in the scheme of [1]. The effects have been found to be not very large; nevertheless they are not negligible for $1\ (\mathrm{GeV}/c)^2 \lesssim Q^2 \lesssim 5\ (\mathrm{GeV}/c)^2$ and could constitute a non-negligible source of error in a high precision determination of α_s from DIS including data in this region.

Acknowledgements

We are grateful to S. Forte and R. Ball for many useful discussions and comments.

References

[1] R. D. Ball and S. Forte, *Nucl. Phys.* **B 425** (1994) 516.

[2] R. D. Ball, V. Barone, S. Forte and M. Genovese, *Phys. Lett.* **B329** (1994) 505.

[3] S. Forte and R. Ball, *Nucl. Phys.* **39 B,C** (Proc. Supl.) (1995) 28; QCD 94 Montpellier, June 1994 ;
M. Genovese, 1994 PhD thesis, 'Some aspects of nucleon structure phenomenology: the deep inelastic scattering as a test bed of QCD', Università di Torino in press.

[4] See for example 'Review of Particle Properties', *Phys. Rev.* **D 50** (1994), vol. 3, part I and references therein.

[5] G. Parisi, *Phys. Lett.* **B50** (1974) 367; G. Parisi, *Proc.* 11^{th} *Rencontre de Moriond*, ed. J. Tran Thanh Van, ed. Frontières, 1976; G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298 ;
See also: Yu.L. Dokshitser, *Sov. Phys. JETP* **46** (1977) 641 ;
V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 438; L.N. Lipatov, *Sov. J. Nucl. Phys.* **20** (1974) 181.

[6] M. Genovese, D. B. Lichtenberg and E. Predazzi, *Z. Phys.* **C61** (1994) 425 and references therein.

[7] G. 't Hooft, *Phys. Rep.* **142** (1986) 357 and references therein.

[8] P. Amaudruz *et al.*, *Phys. Rev.* **D 50** (1994) R1.

[9] V. Barone, M. Genovese, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, *Z. Phys.* **C58** (1993) 541; *Int. J. Mod. Phys.* **A8** (1993) 2779.

[10] See for example: R. G. Roberts, 'The structure of the proton', Cambridge University Press (1990).

[11] H. L. Lai *et al.*, CTEQ-404 (1994).

[12] P. Amaudruz *et al.*, *Nucl. Phys.* **B 371** (1992) 3.

Figure Captions

Fig. 1. The ratio $\frac{d\log F_2}{d\log Q^2}$ obtained with (solid curve) and without (dashed curve) mesonic effects at $Q^2 = 1.6(\mathrm{GeV}/c)^2$. The value $\Lambda_{LO}^{nf=3} = 200$ MeV/c is used.